Chapter 9
Hypothesis Tests About the Mean and Proportion

This chapter introduces the second topic in inferential statistics: tests of hypotheses. In a test of hypothesis, we test a certain given theory or belief about a population parameter. We may want to find out, using some sample information, whether or not a given claim (or statement) about a population parameter is true. This chapter discusses how to make such tests of hypotheses about the population mean, μ, and the population proportion, *p*.

Hypothesis Tests: An Introduction

A **hypothesis test** is a statistical **test** that is used to determine **whether there is enough evidence in a sample of data to infer that a certain condition is true for the entire population.**

We perform a test of hypothesis **only when we are making a decision about a population parameter based on the value of a sample statistic**.

Hypothesis testing involves forming two hypotheses: a null hypothesis and an alternative hypothesis.

Two Hypotheses

A hypothesis test examines two opposing hypotheses about a population: the null hypothesis and the alternative hypothesis. How you set up these hypotheses depends on what you are trying to show.

Null hypothesis (H_0)

The null hypothesis states that a population parameter is = to, or ≤, or ≥ a value. The null hypothesis is often an initial claim that researchers specify using previous research or knowledge.

Alternative Hypothesis (H_1)

The alternative hypothesis states that the population parameter is different than the value of the population parameter in the null hypothesis. The alternative hypothesis is what you might believe to be true or hope to prove true.

Definition

Null Hypothesis A *null hypothesis* is a claim (or statement) about a population parameter that is assumed to be true until it is declared false.

Definition

Alternative Hypothesis An *alternative hypothesis* is a claim about a population parameter that will be declared true if the null hypothesis is declared to be false.

Two Types of Errors

When you do a hypothesis test, two types of errors are possible: type I and type II. **The risks of these two errors are inversely related and determined by the level of significance and the power for the test. Therefore, you should determine which error has more severe consequences for your situation before you define their risks.**

No hypothesis test is 100% certain. Because the test is based on probabilities, there is always a chance of drawing an incorrect conclusion.

Type I error

- **When the null hypothesis is true and you reject it, you make a type I error.**
- The probability of making a type I error is **α, which is the level of significance you set for your hypothesis test.**
- An α of 0.05 indicates that you are willing to accept a 5% chance that you are wrong when you reject the null hypothesis.

Type II error

- **When the null hypothesis is false and you fail to reject it, you make a type II error.**
- The probability of making a type II error is β.
- The probability of rejecting the null hypothesis when it is false is equal to **$1–\beta$. This value is the power of the test.**

	Null Hypothesis	
Decision	True	False
Fail to reject	Correct Decision (probability = 1 - α)	**Type II Error** - fail to reject the null when it is false (probability = β)
Reject	**Type I Error** - rejecting the null when it is true (probability = α)	Correct Decision (probability = 1 - β)

Tails of a Test

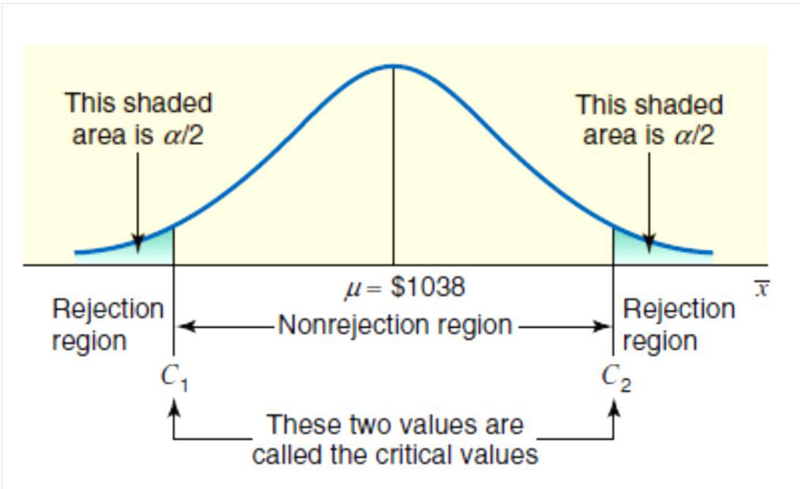
A test with two rejection regions is called a **two-tailed test**, and a test with one rejection region is called a **onetailed test**. The one-tailed test is called a **left-tailed test** if the rejection region is in the left tail of the distribution curve, and it is called a **right-tailed test** if the rejection region is in the right tail of the distribution curve.

Table 9.3 Signs in H_0 and H_1 and Tails of a Test

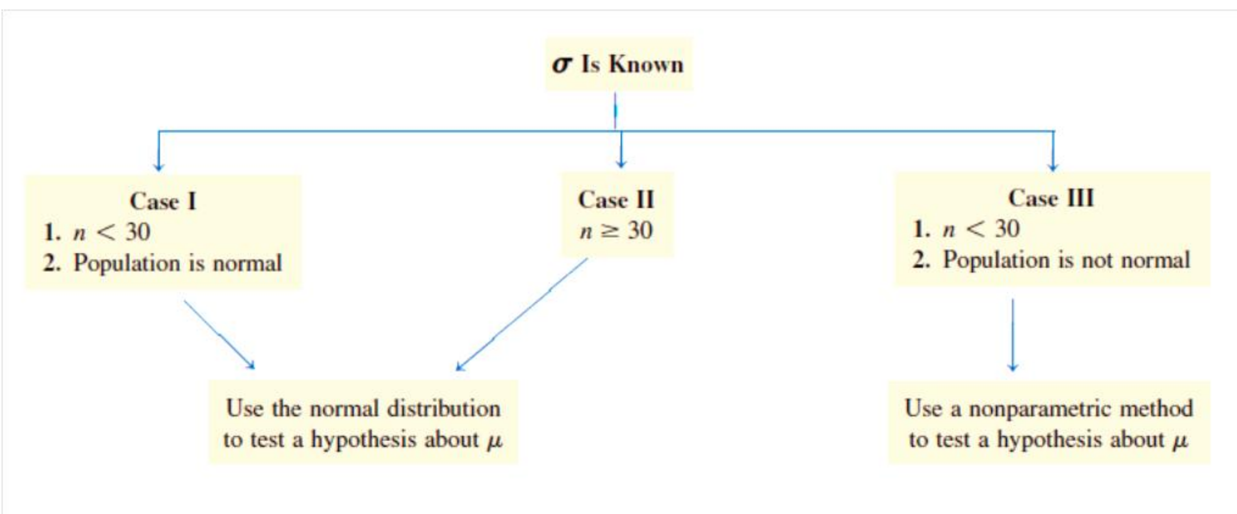
	Two-Tailed Test	Left-Tailed Test	Right-Tailed Test
Sign in the null hypothesis H_0	$=$	$=$ or $\geq$	$=$ or $\leq$
Sign in the alternative hypothesis H_1	$\neq$	$<$	$>$
Rejection region	In both tails	In the left tail	In the right tail

Note that the null hypothesis always has an *equal to* ($=$) or a *greater than or equal to* ($\geq$) or a *less than or equal to* ($\leq$) sign, and the alternative hypothesis always has a *not equal to* ($\neq$) or a *less than* ($<$) or a *greater than* ($>$) sign.

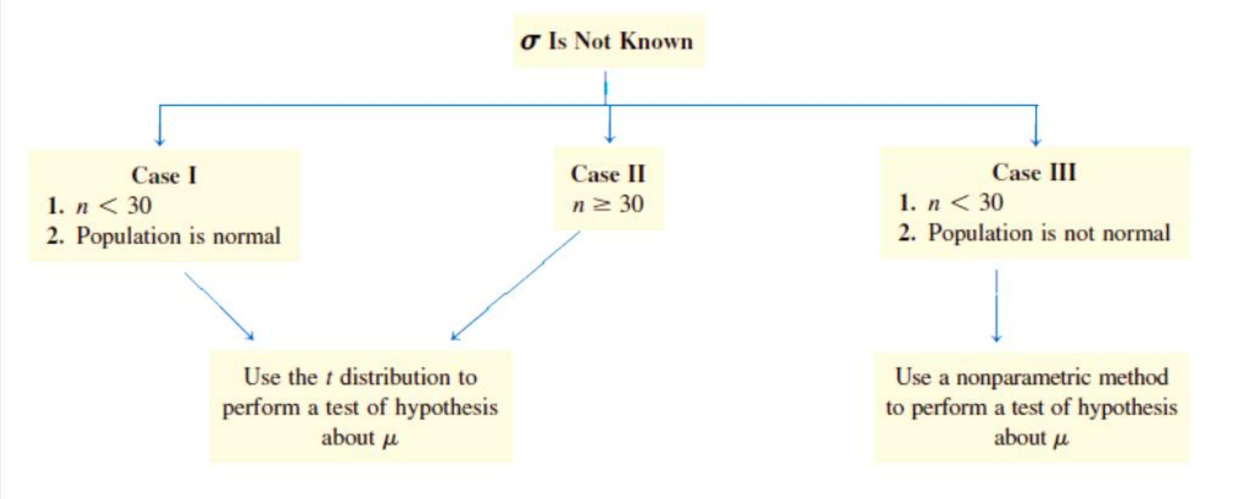
When performing a two-tailed test, remember to divide the value of α by 2.



Hypothesis Tests About μ: σ Known



Hypothesis Tests About μ: σ Not Known



Below we discuss Cases I and II and learn how to use the t distribution to perform a test of hypothesis about μ when σ is not known. When the conditions mentioned for Case I or Case II are satisfied, the random variable

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} \qquad \text{where} \qquad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

has a t distribution. Here, the t is called the **test statistic** to perform a test of hypothesis about a population mean μ.

The *p*-Value Approach

Steps to Perform a Test of Hypothesis Using the *p*-Value Approach

1. State the null and alternative hypotheses.
2. Select the distribution to use.
3. Calculate the p-value.
4. Make a decision.

Using statistical software is the way to go when using the p-value approach, because using the t-distribution table we can only find a range of p-values whereas using technology we can calculate the exact p-value.

The Critical-Value Approach

Steps to Perform a Test of Hypothesis with the Critical-Value Approach

1. State the null and alternative hypotheses.
2. Select the distribution to use.
3. Determine the rejection and nonrejection regions.
4. Calculate the value of the test statistic.
5. Make a decision.

We will use the critical value approach for solving the problems involving hypothesis testing about μ when σ is not known.

Solve: 9.60, 9.61, 9.64, 9.70, 9.66

Hypothesis Tests About a Population Proportion: Large Samples

This section presents the procedure to perform tests of hypotheses about the population proportion, p, for large samples. The procedures to make such tests are similar in many respects to the ones for the population mean, μ. Again, the test can be two-tailed or one-tailed. We know from Chapter 7 that when the sample size is large, the sample proportion, $\hat{p}$, is approximately normally distributed with its mean equal to p and standard deviation equal to $\sqrt{pq/n}$. Hence, we use the normal distribution to perform a test of hypothesis about the population proportion, p, for a large sample. As was mentioned in Chapters 7 and 8, in the case of a proportion, the sample size is considered to be large when np and nq are both greater than 5.

Test Statistic The value of the *test statistic* z for the sample proportion, $\hat{p}$, is computed as

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} \quad \text{where} \quad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

The value of p that is used in this formula is the one from the null hypothesis. The value of q is equal to $1 - p$.

The value of z calculated for $\hat{p}$ using the above formula is also called the **observed value of** z.

- Both p-value approach and critical value approach can be used to hypothesis about population proportion.

- **In this case, we can get the exact value of p using the standard normal distribution or z distribution table.**

- Both approaches will give the same result of course, but using critical-value approach would require one additional step.

So we will use the p-value approach for solving the problems involving hypothesis testing about population proportions.

Solve: 9.88, 9.89, 9.90, 9.92, 9.93

Additional practice problems: 9.65, 9.71, 9.91, 9.94 (from 8th edition of the book)